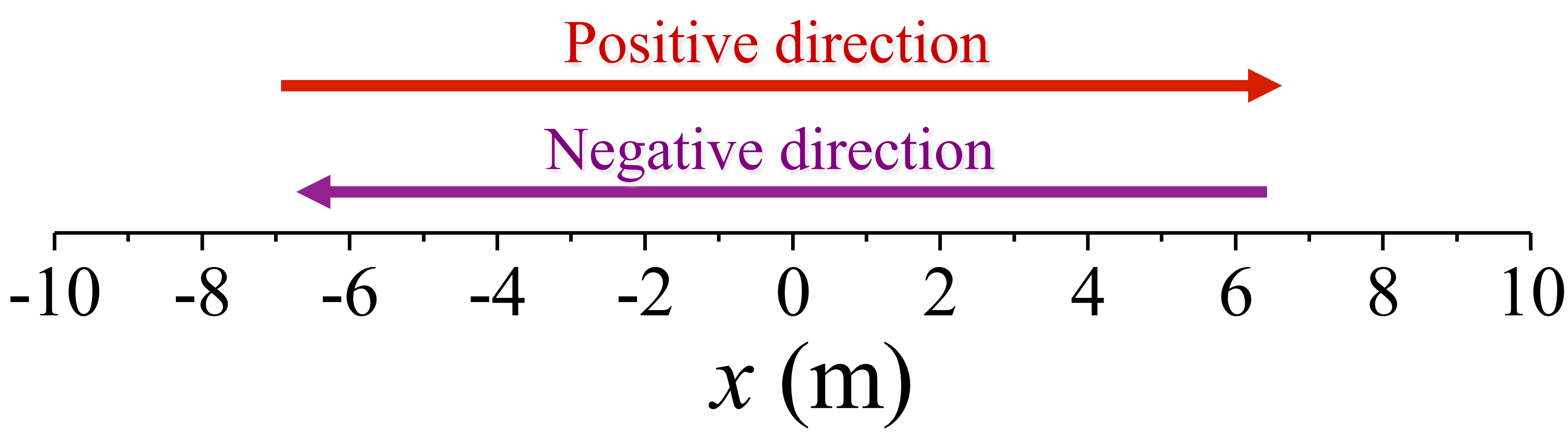
Chapter 2: 1D Kinematics
Tuesday January 13th

- Motion in a straight line (1D Kinematics)
  - Average velocity and average speed
  - Instantaneous velocity and speed
  - Acceleration
  - Short summary
- Constant acceleration – a special case
  - Free-fall acceleration

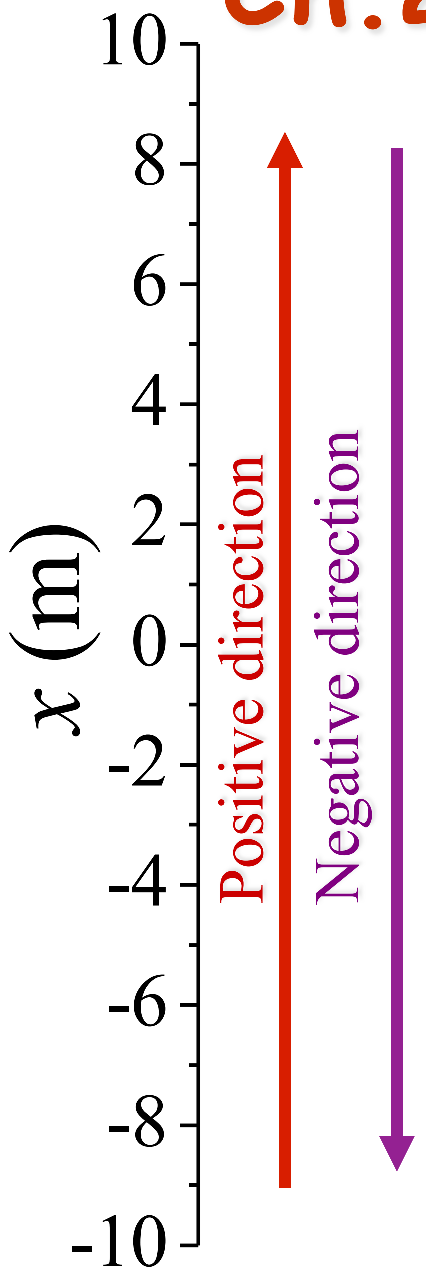
Reading: up to page 25 in the text book (Ch. 2)

Ch.2: Motion in one-dimension

- We will define the position of an object using the variable x, which measures the position of the object relative to some reference point (origin) along a straight line (x-axis).

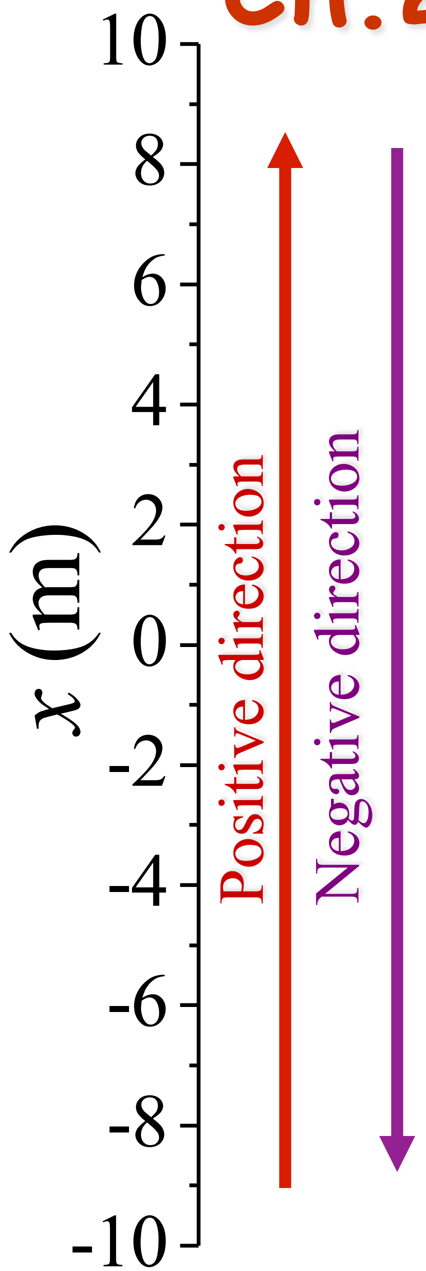


Ch.2: Motion in one-dimension



- We will define the position of an object using the variable x, which measures the position of the object relative to some reference point (origin) along a straight line (x-axis).

Ch.2: Motion in one-dimension



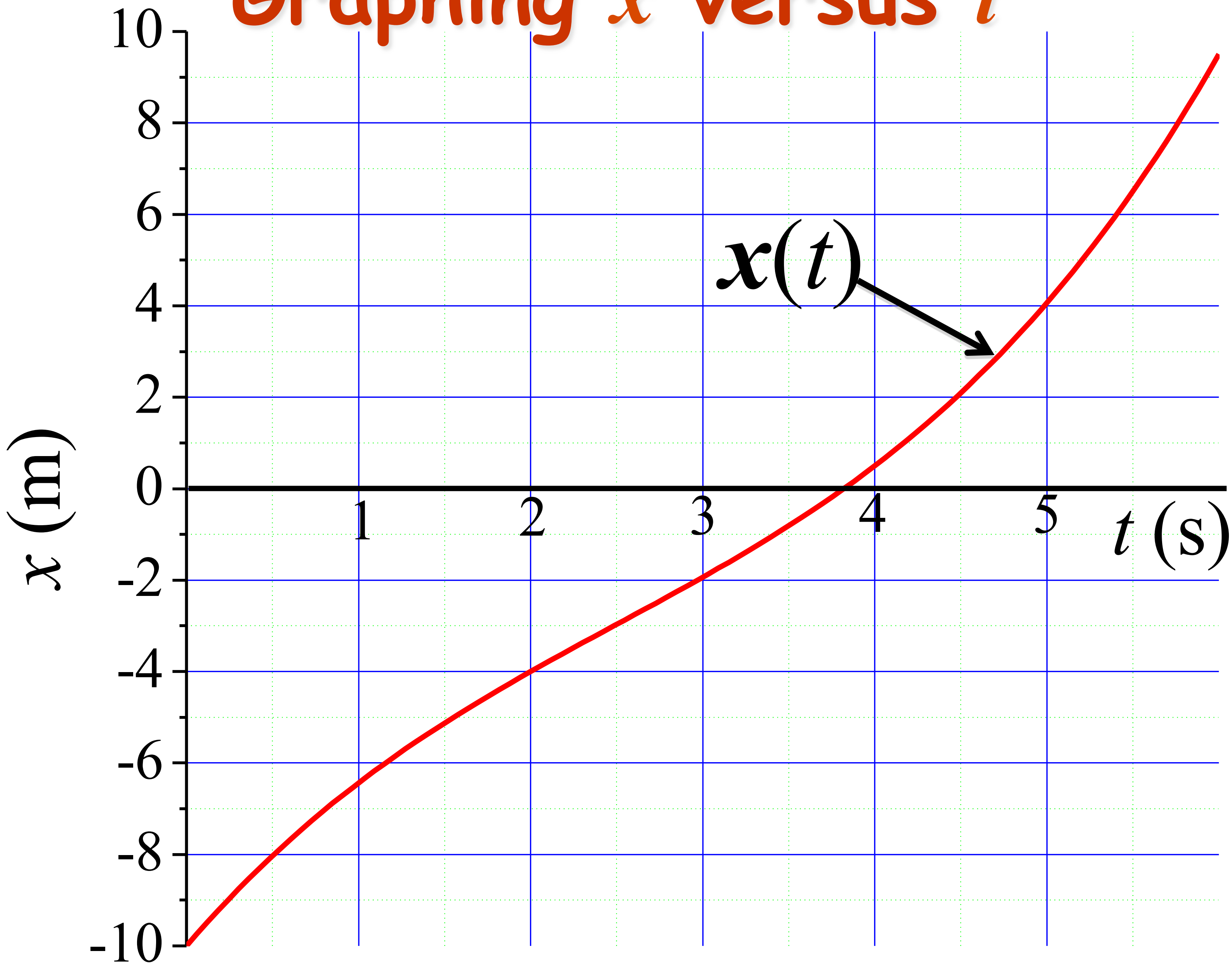
- We will define the position of an object using the variable x, which measures the position of the object relative to some reference point (origin) along a straight line (x-axis).

- In general, x will depend on time t.

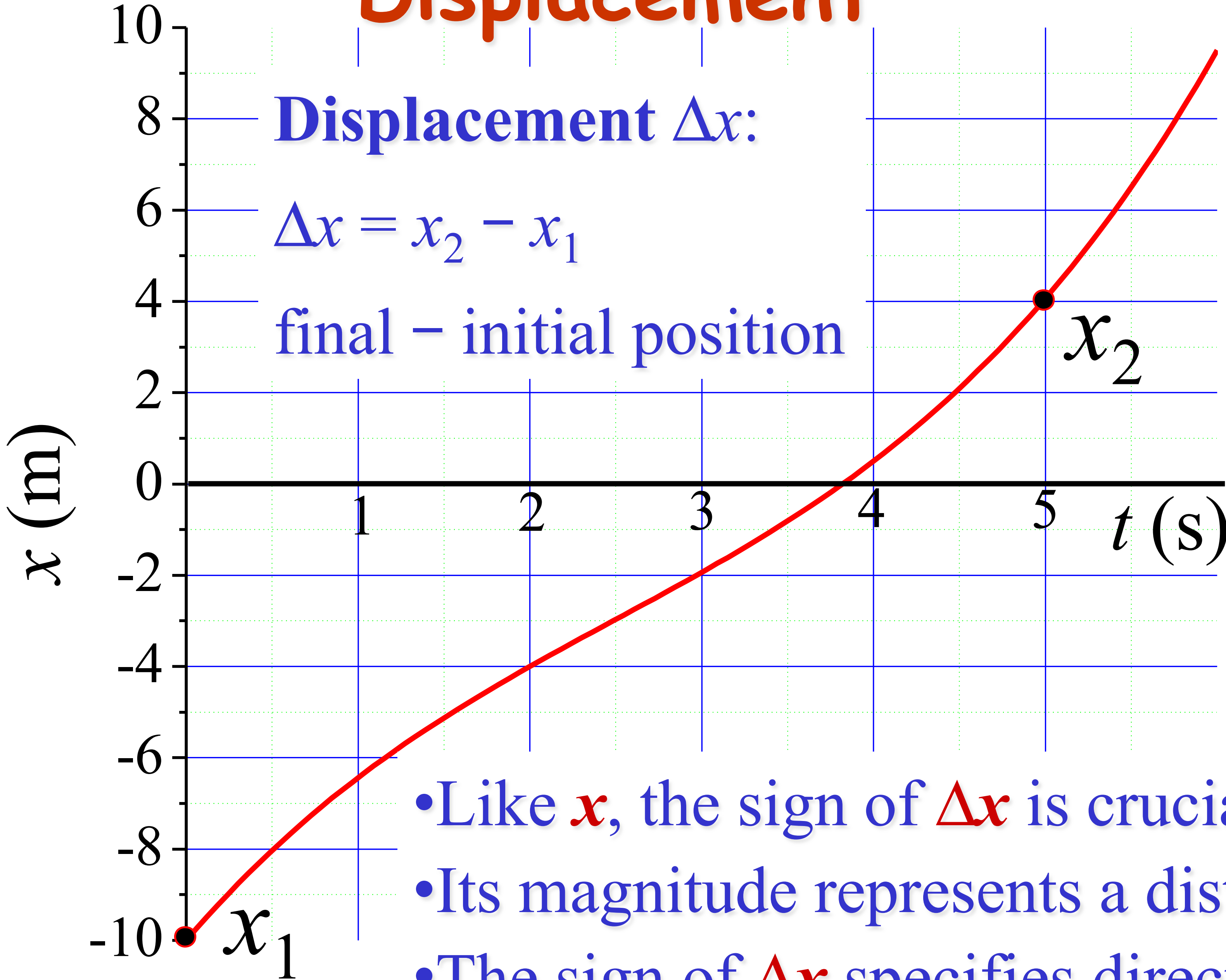
- We shall measure x in meters, and t in seconds, *i.e.* SI units.

- Although we will only consider only one-dimensional motion here, we should not forget that x is a component of a vector. Thus, motion in the $+x$ and $-x$ directions correspond to motions in opposite directions.

Graphing x versus t



Displacement



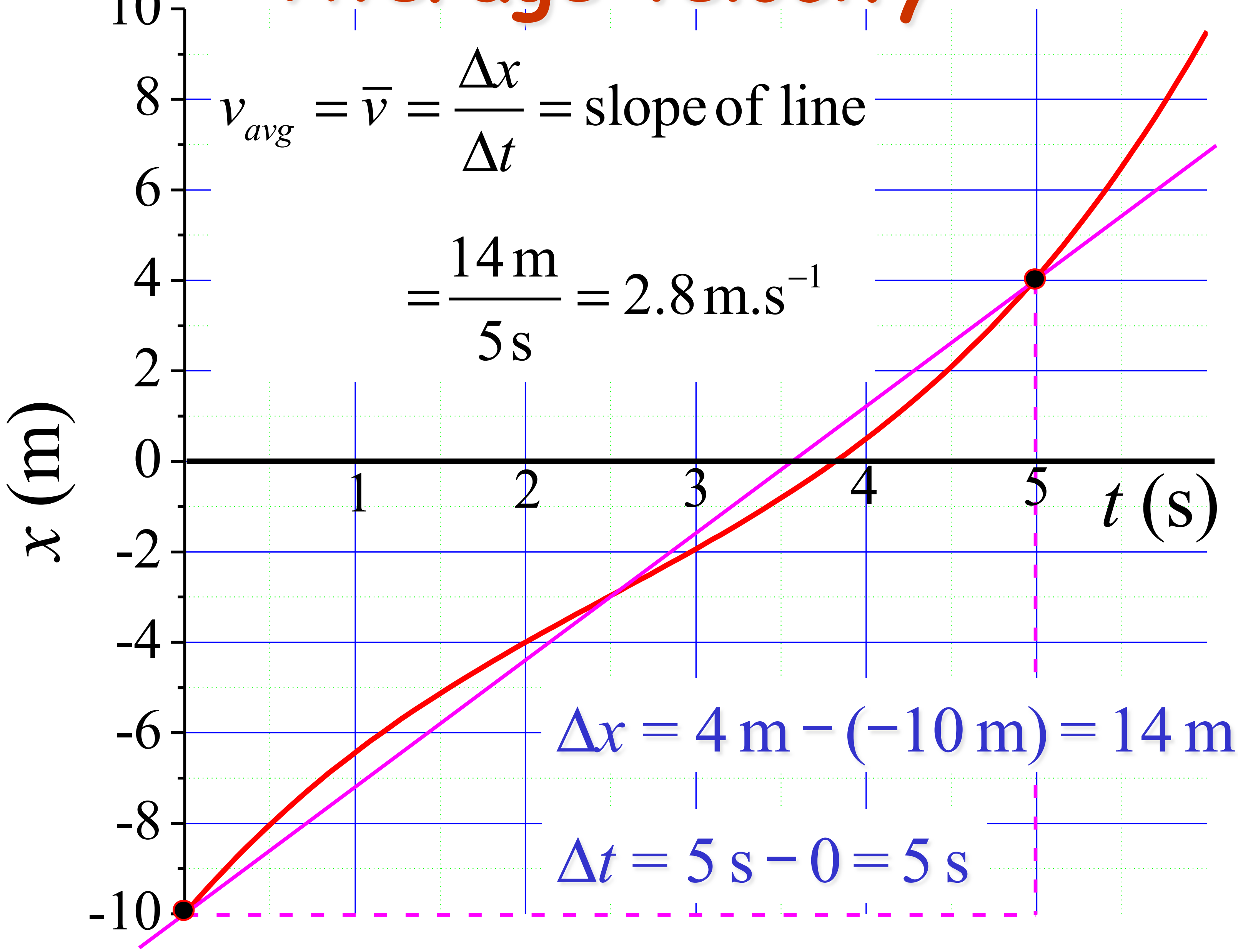
Displacement Δx:

$\Delta x = x_2 - x_1$

final − initial position

- Like ***x***, the sign of **Δ*x*** is crucial
- Its magnitude represents a distance
- The sign of **Δ*x*** specifies direction

Average velocity



Average velocity and speed

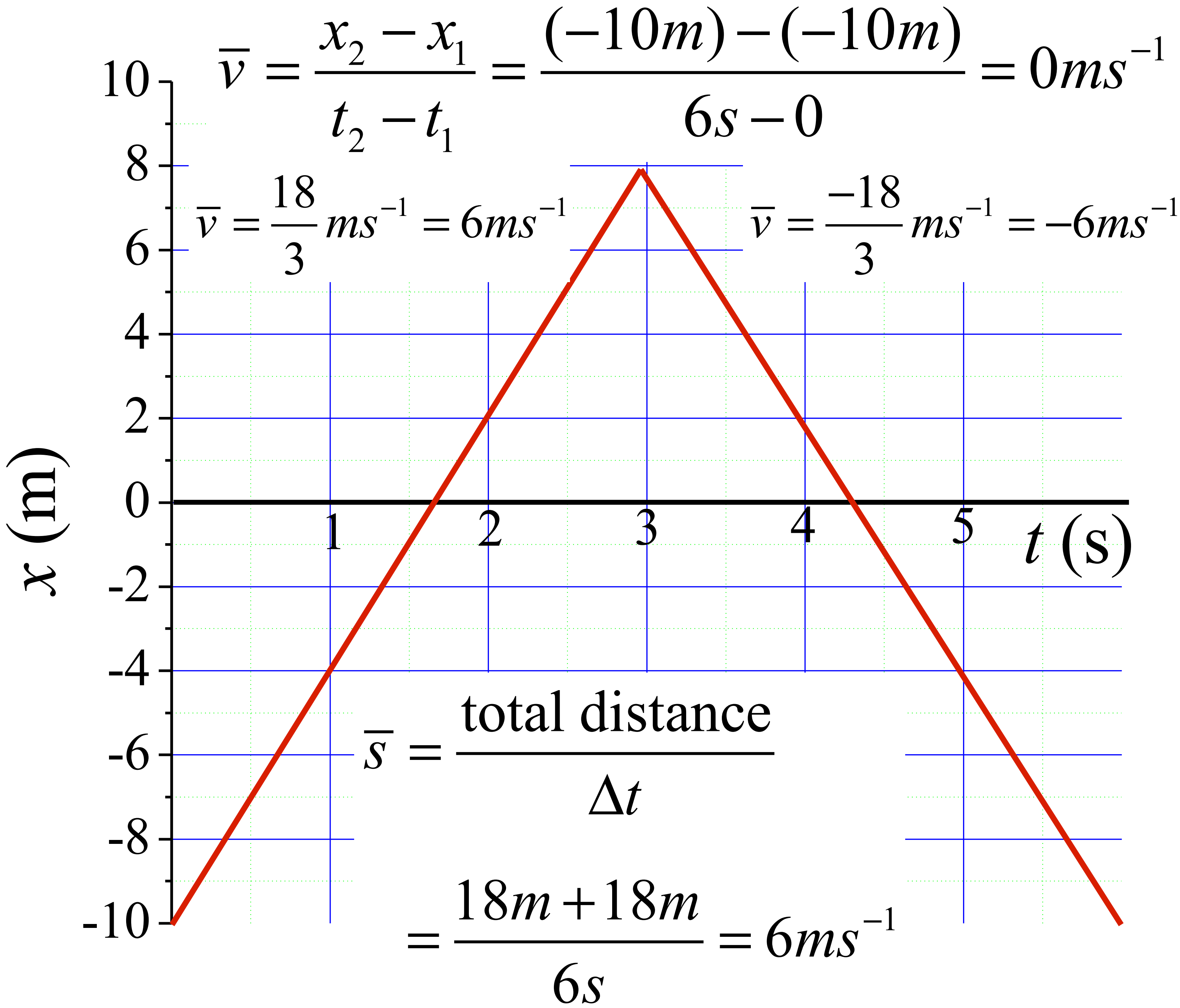
$$v_{avg} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- Like displacement, the sign of v_{avg} indicates direction

Average speed s_{avg}:

$$s_{avg} = \bar{s} = \frac{\text{total distance}}{\Delta t}$$

- s_{avg} does not specify a direction; it is a scalar as opposed to a vector &, thus, lacks an algebraic sign
- How do v_{avg} and s_{avg} differ?



$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(-10m) - (-10m)}{6s - 0} = 0ms^{-1}$$

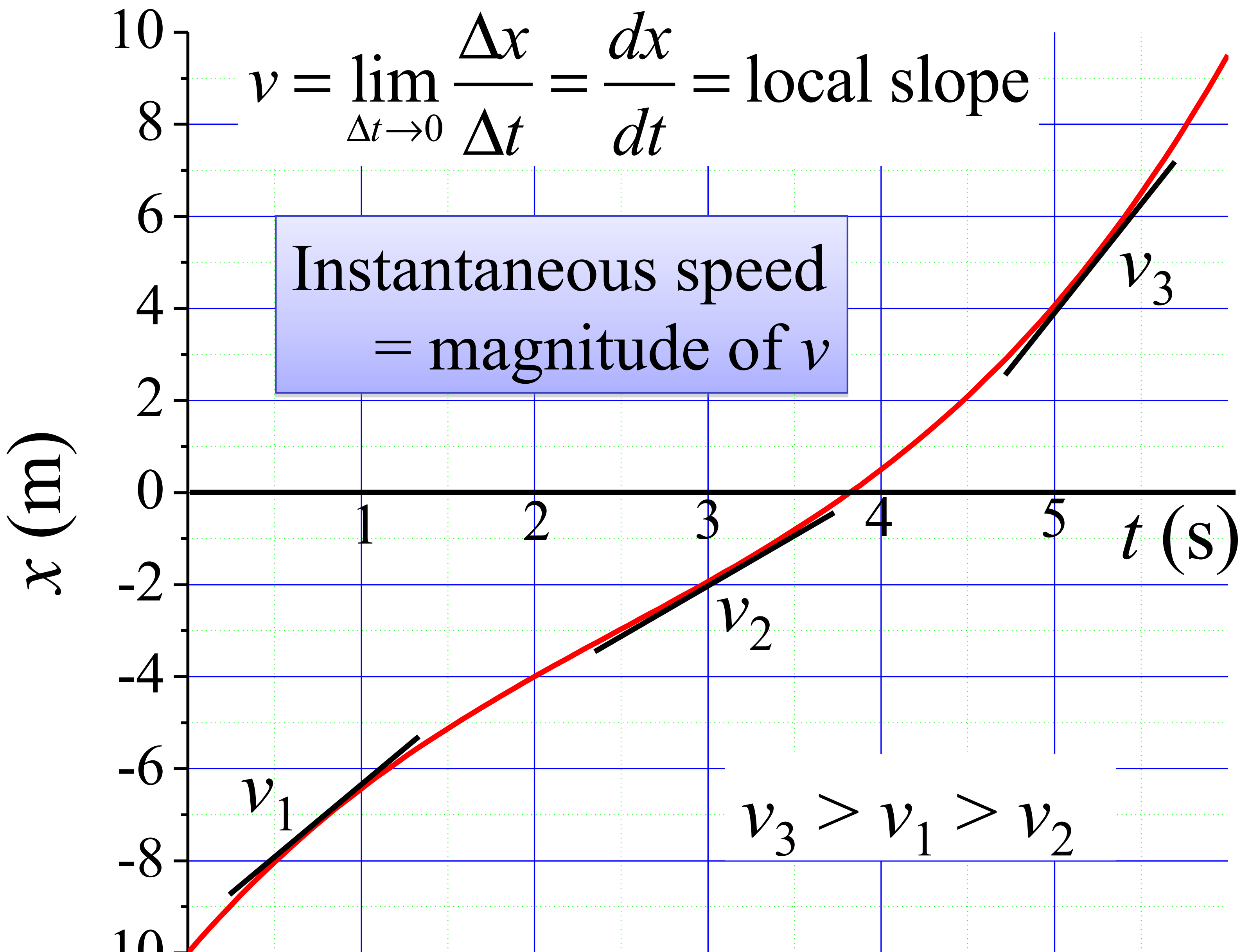
$$\bar{v} = \frac{18}{3} ms^{-1} = 6ms^{-1}$$

$$\bar{v} = \frac{-18}{3} ms^{-1} = -6ms^{-1}$$

$$\bar{s} = \frac{\text{total distance}}{\Delta t}$$

$$= \frac{18m + 18m}{6s} = 6ms^{-1}$$

Instantaneous velocity and speed



Acceleration

- An object is accelerating if its velocity is changing

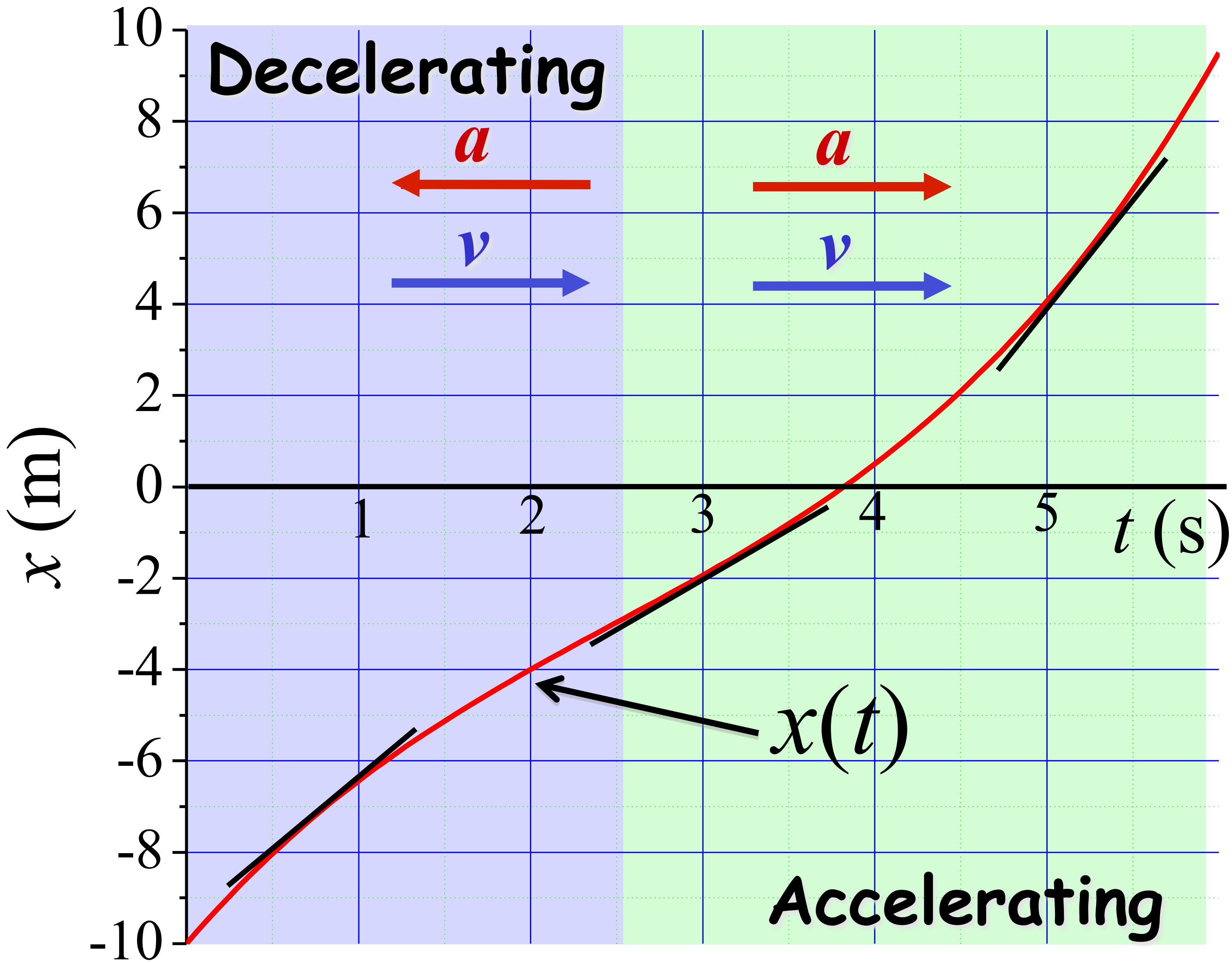
Average acceleration a_{avg}**:**

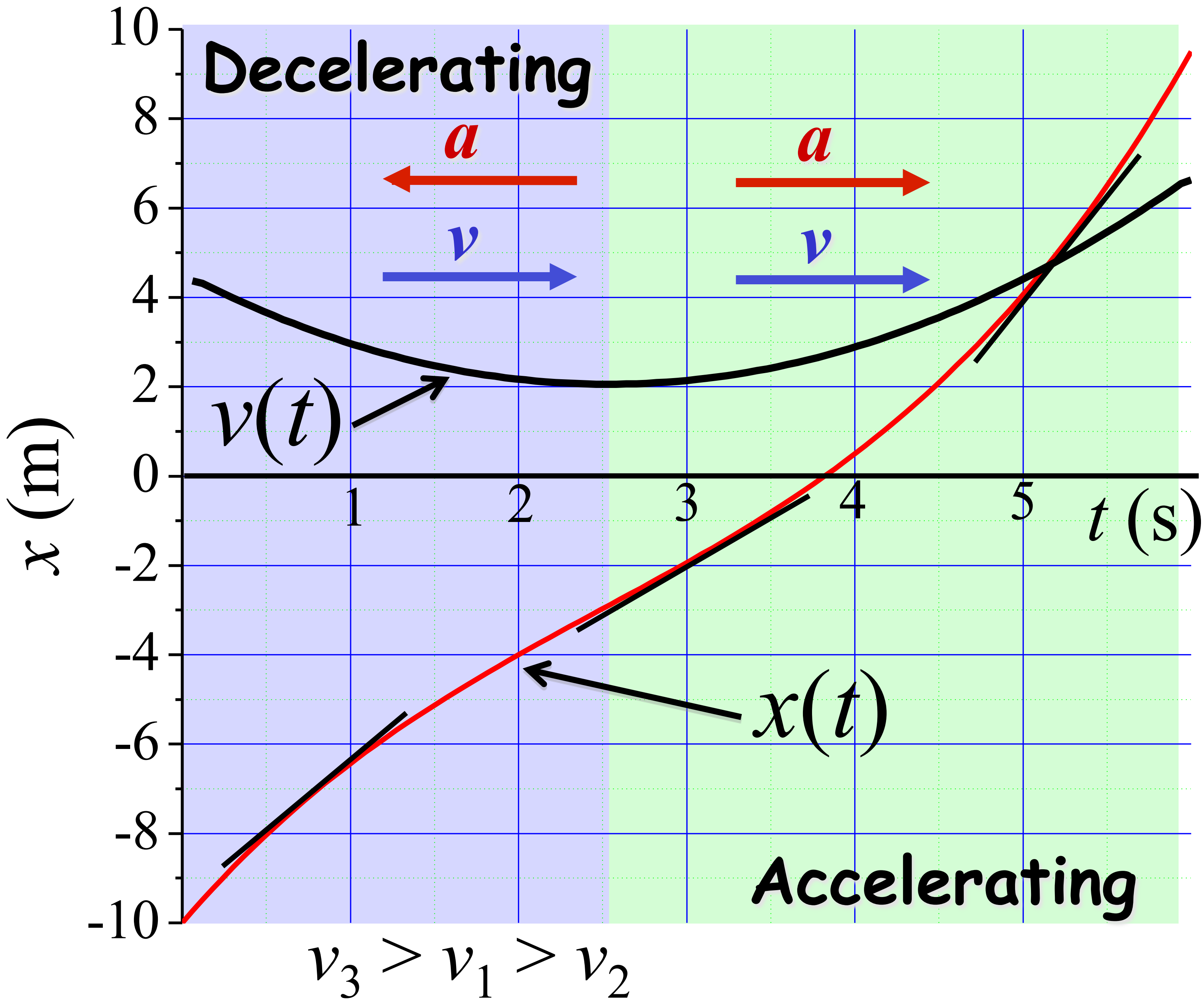
$$a_{avg} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration a**:**

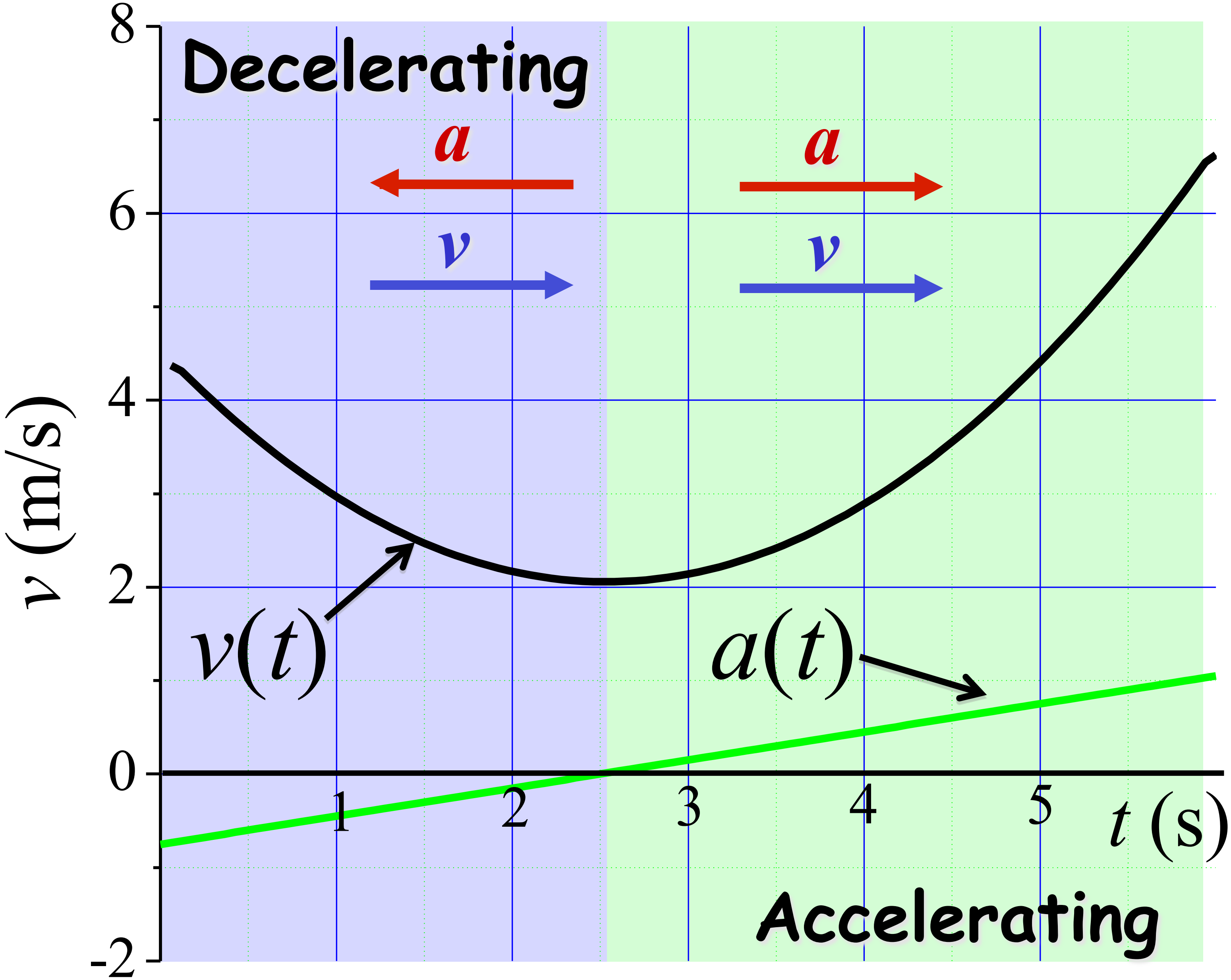
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

- This is the second derivative of the x vs. t graph
- Like x and v, acceleration is a vector
- Note: direction of a need not be the same as v


Decelerating
a
v
a
v
Accelerating
x(t)
x (m)
t (s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
1
2
3
4
5



Decelerating
Accelerating
a
v
a
v
v(t)
x(t)
x (m)
t (s)
10
8
6
4
2
0
-2
-4
-6
-8
-10
1
2
3
4
5
$v_3 > v_1 > v_2$



Decelerating
a
v
a
v
8
6
4
2
0
-2
v (m/s)
v(t)
a(t)
1
2
3
4
5
t (s)
Accelerating

Summarizing

Displacement: $\Delta x = x_2 - x_1$

Average velocity: $v_{avg} = \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1}$

Average speed: $s_{avg} = \bar{s} = \dfrac{\text{total distance}}{\Delta t}$

Instantaneous velocity:

$$v = \frac{dx}{dt} = \text{local slope of } x \text{ versus } t \text{ graph}$$

Instantaneous speed: magnitude of v

Summarizing

Average acceleration: $$a_{avg} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Instantaneous acceleration:

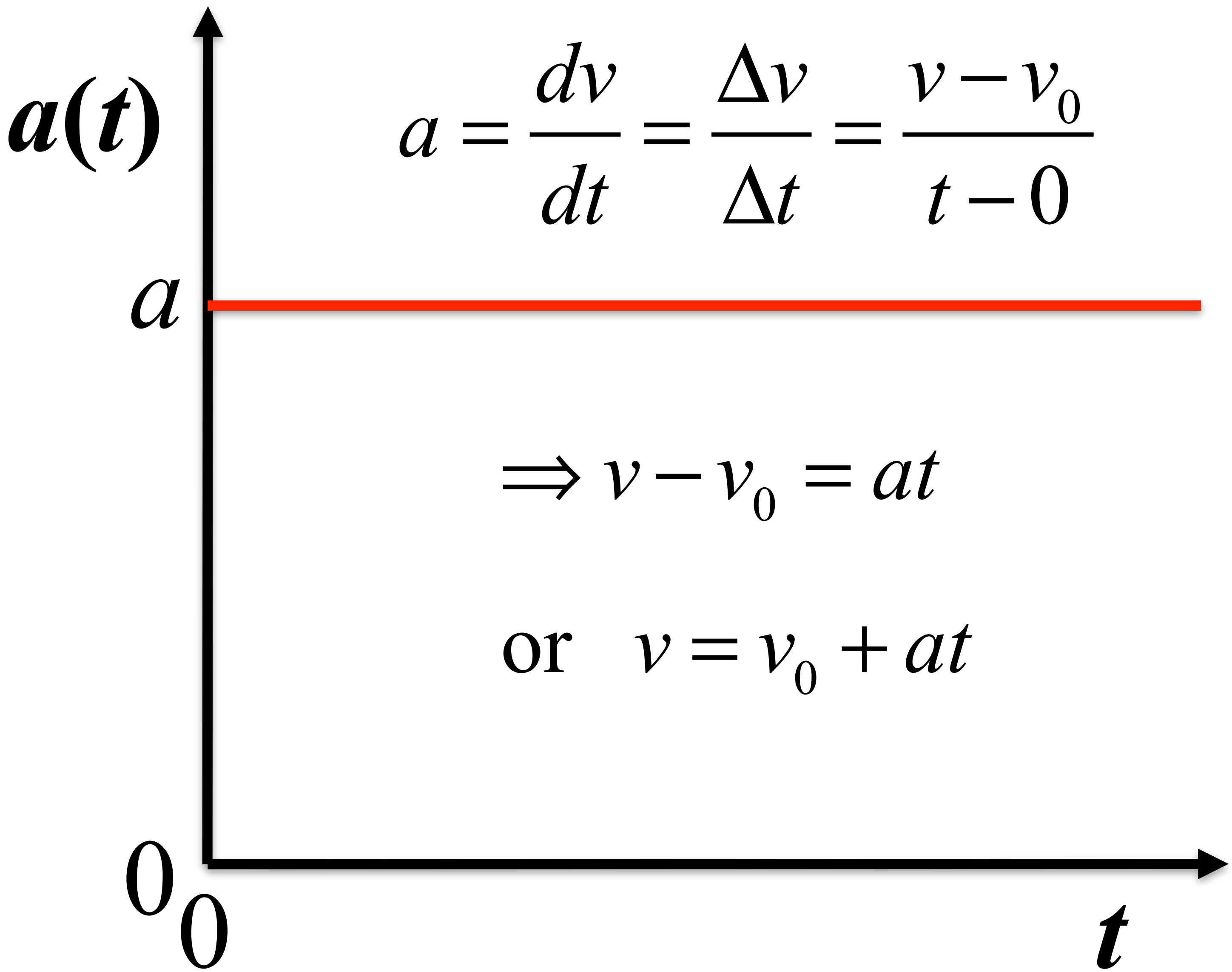
$$a = \frac{dv}{dt} = \text{local slope of } v \text{ versus } t \text{ graph}$$

In addition:

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \text{curvature of } x \text{ versus } t \text{ graph}$$

SI units for a are m/s^2 or $m.s^{-2}$ (ft/min^2 also works)

Constant acceleration: a special case

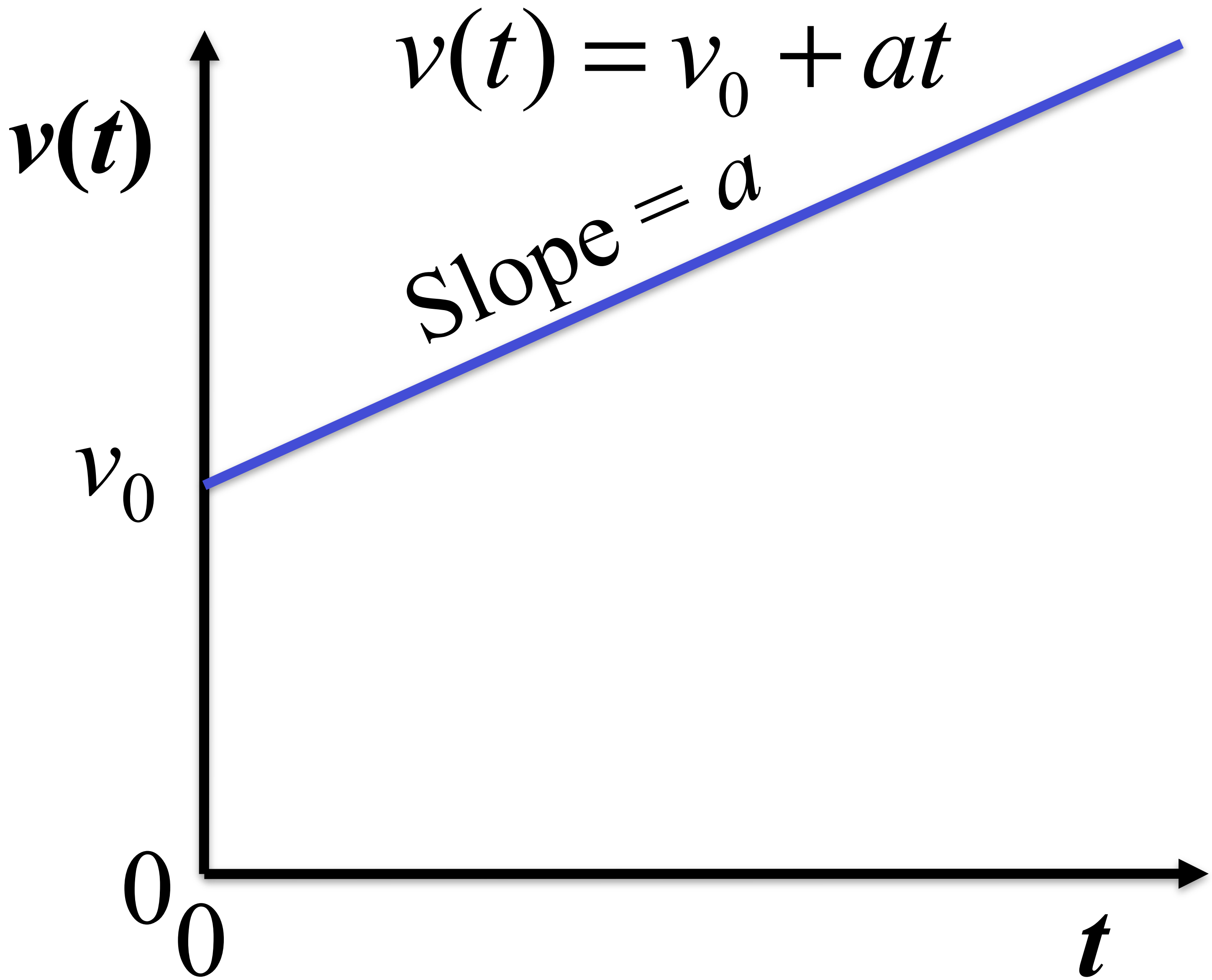


$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$$

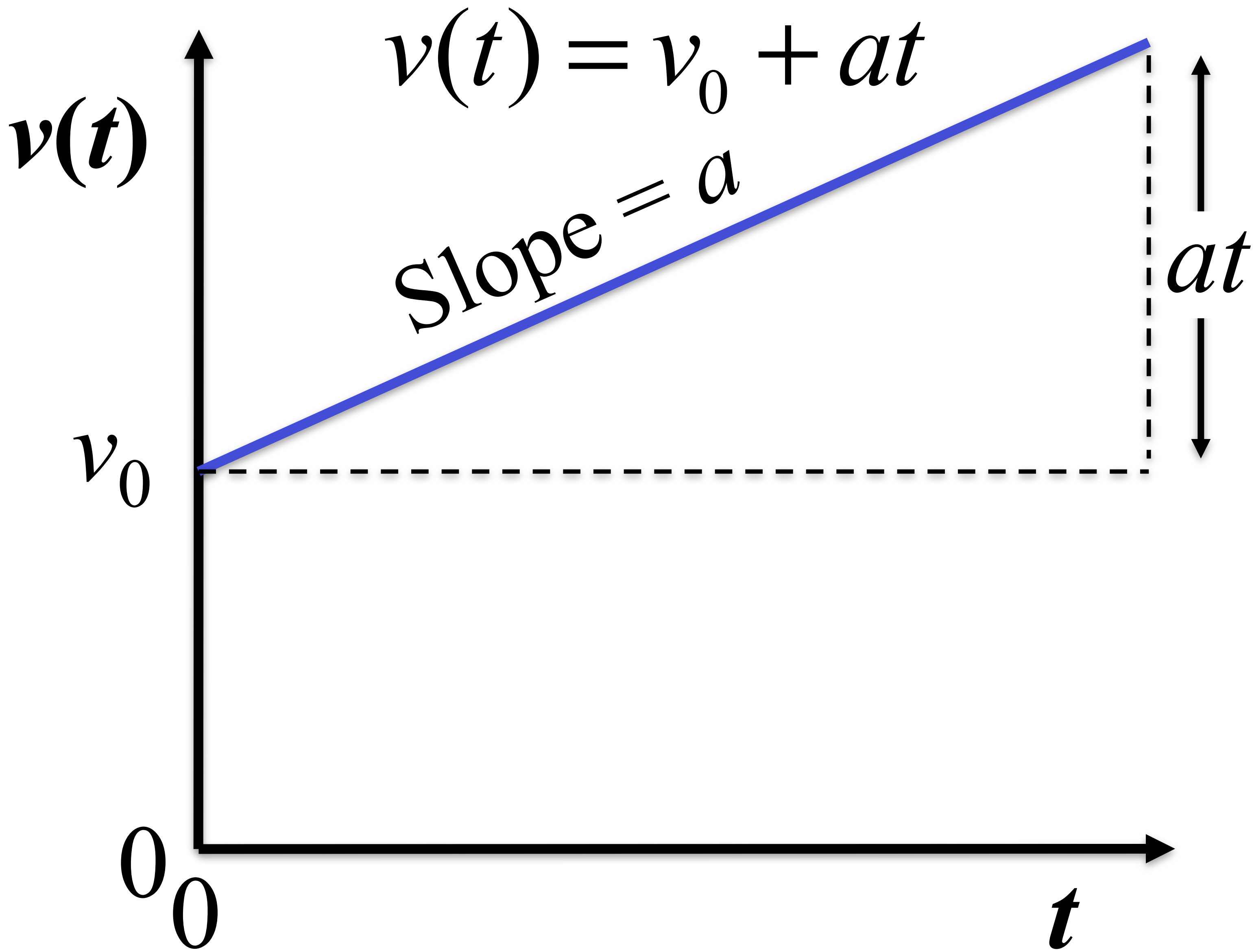
$$\Rightarrow v - v_0 = at$$

$$\text{or} \quad v = v_0 + at$$

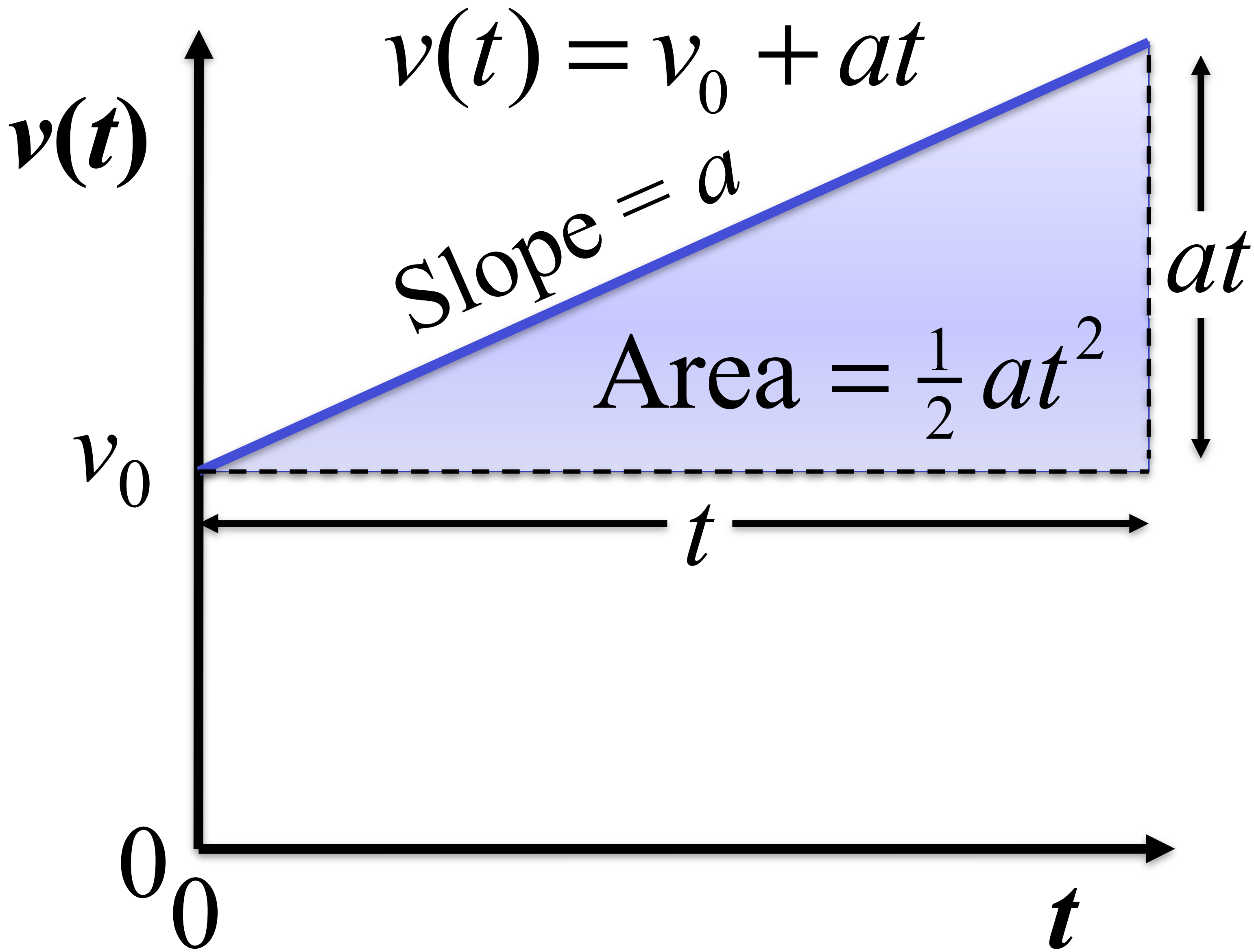
Constant acceleration: a special case



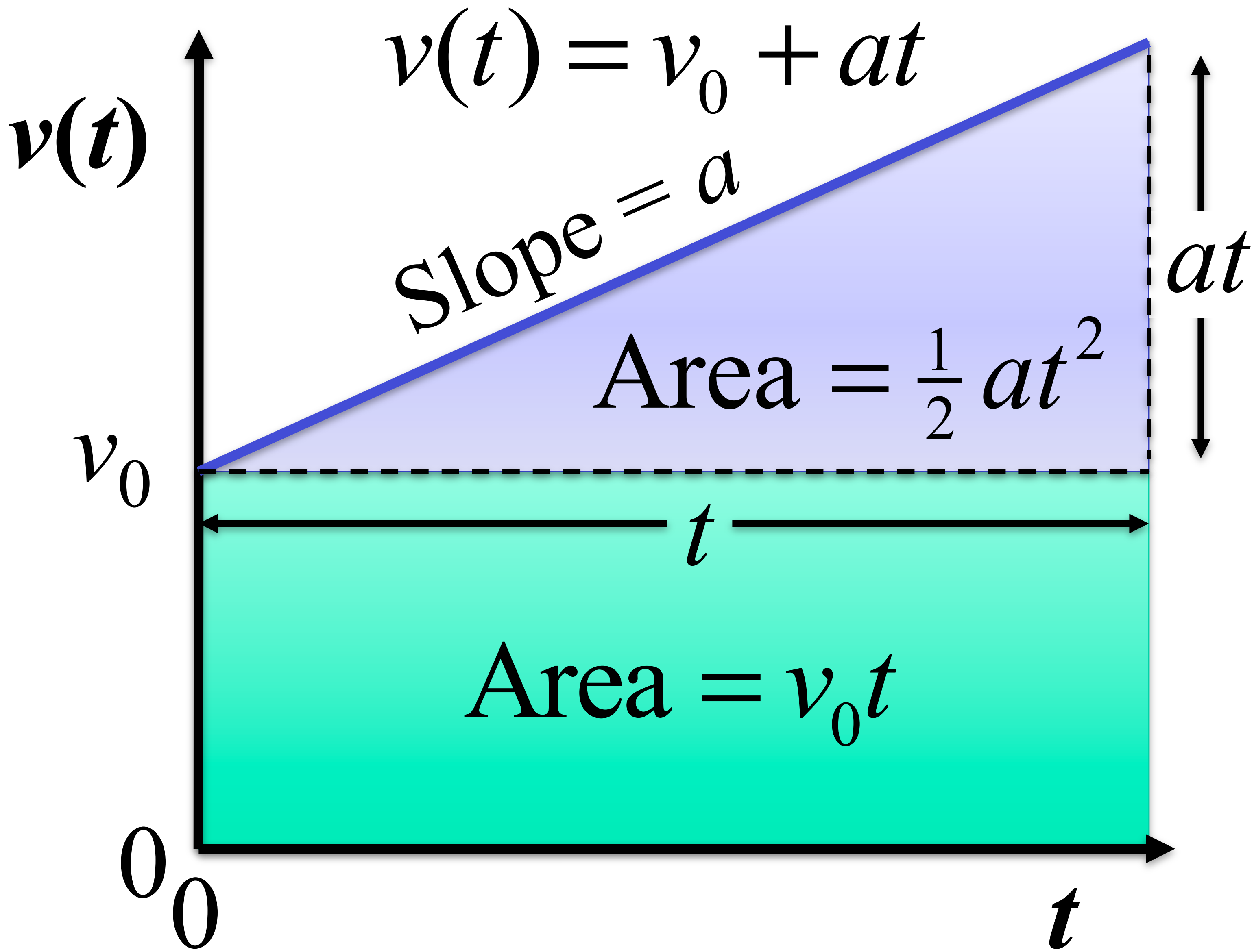
Constant acceleration: a special case



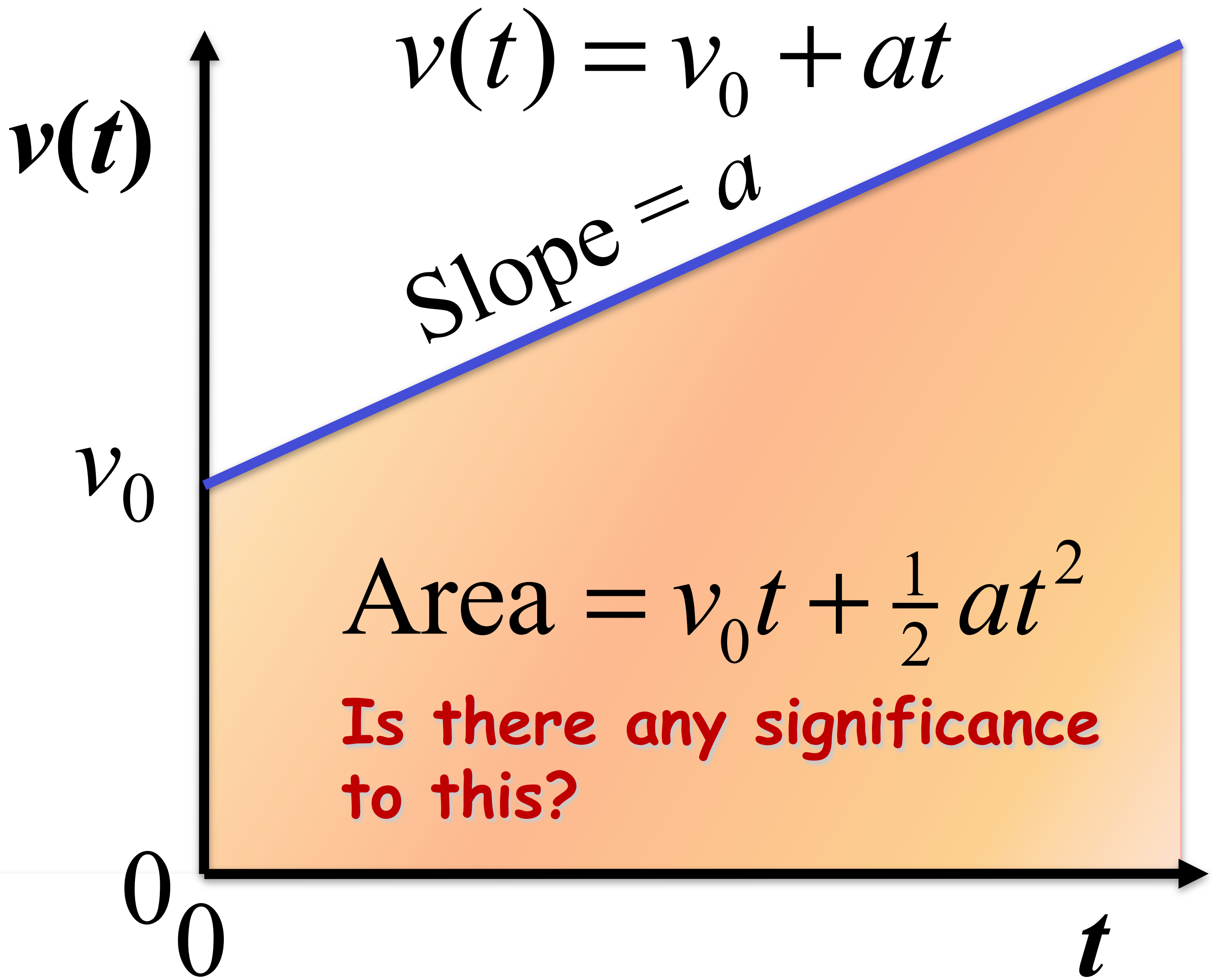
Constant acceleration: a special case



Constant acceleration: a special case

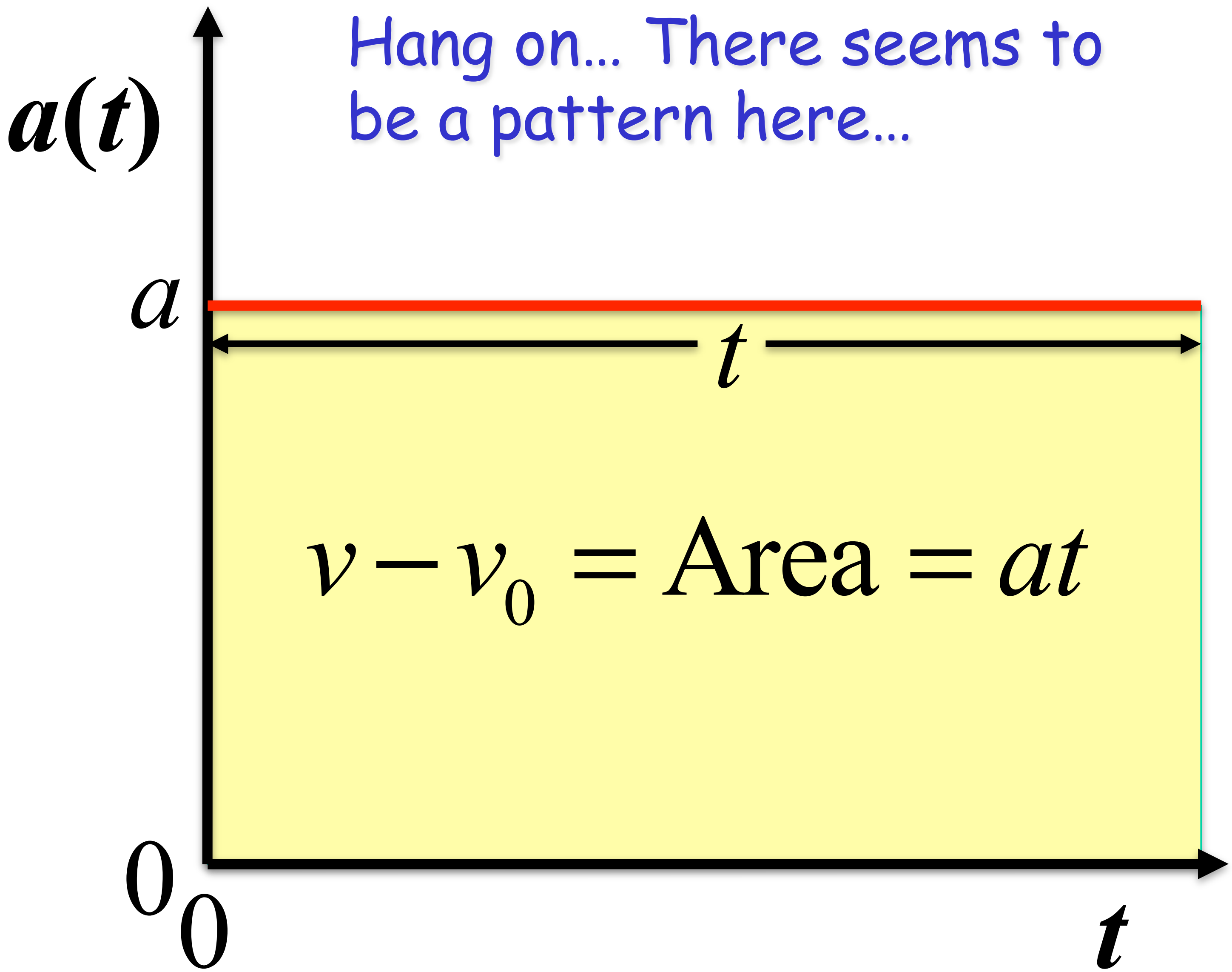


Constant acceleration: a special case



Constant acceleration: a special case

Hang on… There seems to be a pattern here…



$$v - v_0 = \text{Area} = at$$

Constant acceleration: a special case

It is rigorously true (a mathematical fact):

$$v - v_0 = \text{Area under } a(t) \text{ curve} = at$$

$$x - x_0 = \text{Area under } v(t) \text{ curve} = v_0 t + {}^{1}/{}_{2}at^2$$

What we have discovered here is integration or calculus…

$$a(t) = \frac{dv}{dt}$$

$$\Delta v = \int_{v_0}^{v} dv = \int_{0}^{t} a\, dt = \text{Area under curve}$$

$$v - v_0 = at$$

Constant acceleration: a special case

It is rigorously true (a mathematical fact):

$$v - v_0 = \text{Area under } a(t) \text{ curve} = at$$

$$x - x_0 = \text{Area under } v(t) \text{ curve} = v_0 t + {}^1/_2 a t^2$$

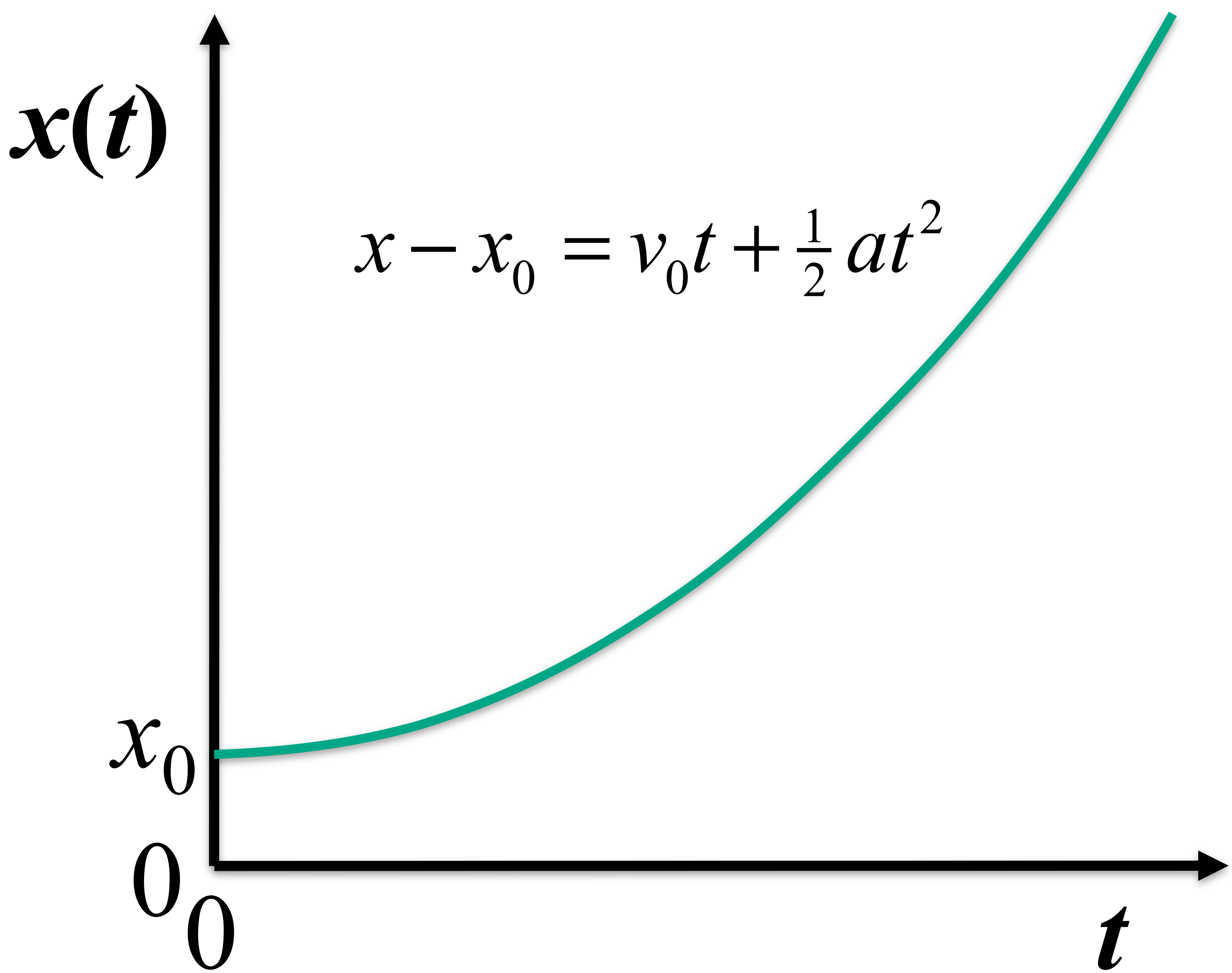
What we have discovered here is integration or calculus…

$$v(t) = \frac{dx}{dt}$$

$$\Delta x = \int_{x_0}^{x} dx = \int_0^t v(t)\,dt = \int_0^t (v_0 + at)\,dt = \text{Area}$$

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

Constant acceleration: a special case



Equations of motion for constant acceleration

One can easily eliminate either a, t or v_o by solving Eqs. 2-7 and 2-10 simultaneously.

Equation number	Equation	Missing quantity
2.7	$v = v_0 + at$	$x - x_0$
2.10	$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
2.11	$v^2 = v_0^2 + 2a(x - x_0)$	t
2.9	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
	$x - x_0 = vt - \frac{1}{2} a t^2$	v_0

Important: equations apply ONLY if acceleration is constant.

A Real Example: Free fall acceleration

- If one eliminates the effects of air resistance, one finds that ALL objects accelerate downwards at the same constant rate at the Earth's surface, regardless of their mass (Galileo).
- That rate is called the free-fall acceleration g.
- The value of g varies slightly with latitude, but for this course g is taken to be $9.81\ \mathrm{ms^{-2}}$ at the earth's surface.
- It is common to consider y as increasing in the upward direction. Therefore, the acceleration a due to gravity is in the negative y direction, i.e. $a_y = -g = -9.8\ \mathrm{ms^{-2}}$.

NOTE: There is nothing special about the parameter *y*. You can use any labels you like, e.g., *x*, *z*, *x*, etc.. The equations we have derived work quite generally.

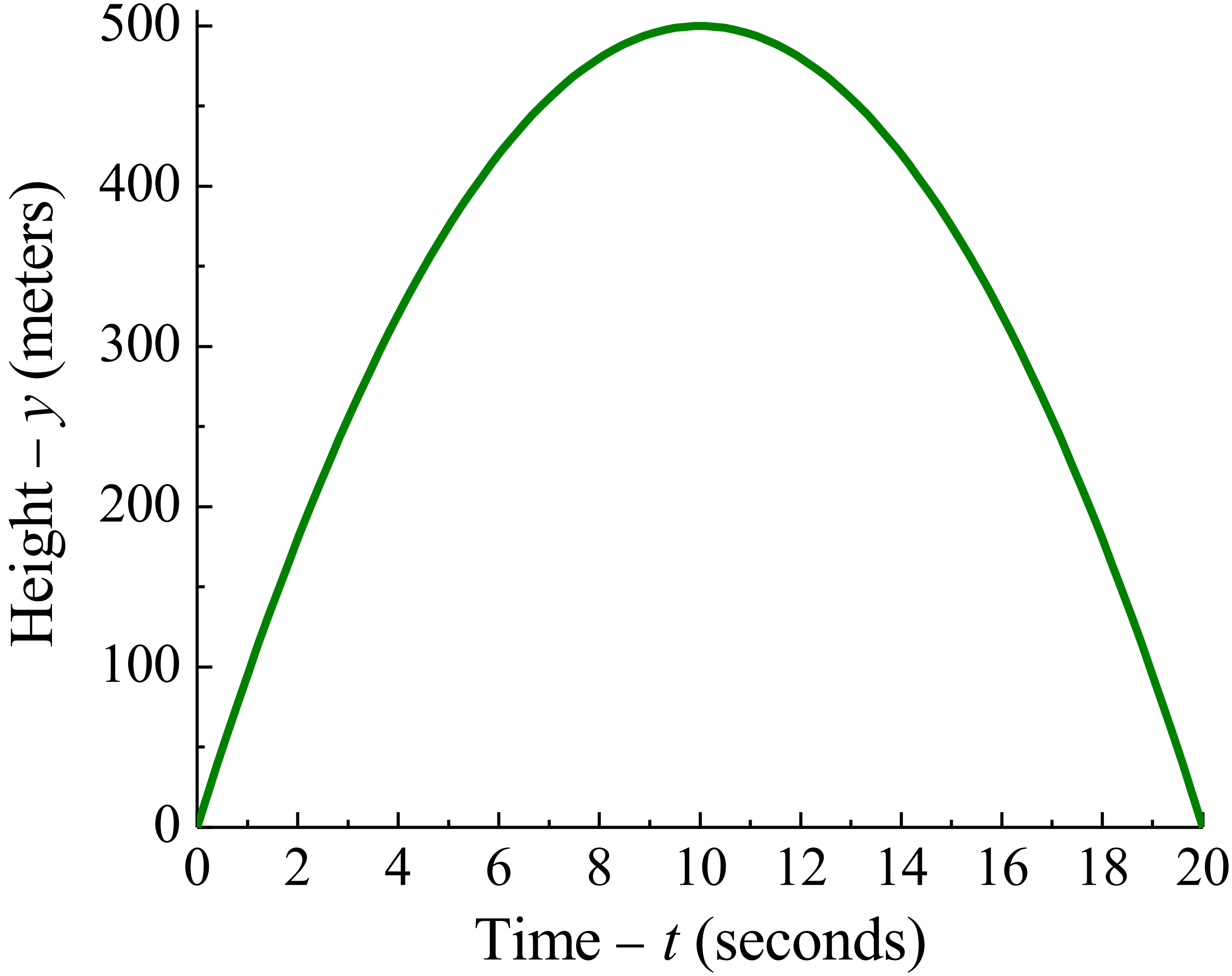
Equations of motion for constant acceleration

One can easily eliminate either a_y, t or v_{oy} by solving Eqs. 2-7 and 2-10 simultaneously.

Equation number	Equation	Missing quantity
2.7	$v_y = v_{0y} + a_y t$	$y - y_0$
2.10	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$	v_y
2.11	$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$	t
2.9	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$	a_y
	$y - y_0 = v_y t - \frac{1}{2} a_y t^2$	v_{0y}

Important: equations apply ONLY if acceleration is constant.

A Real Example: Free fall acceleration



A Real Example: Free fall acceleration